Determination of the Random Error in Level and Staff Measurements

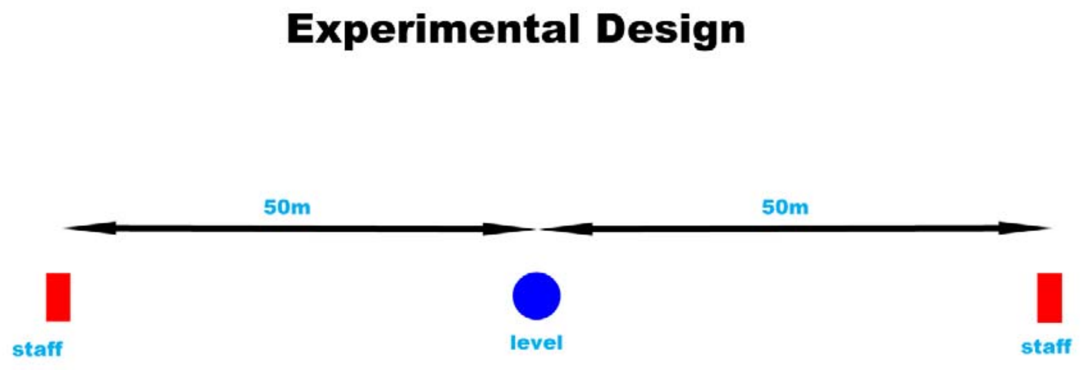
1) Introduction

G&J Surveys uses a Leica NA730 Professional Automatic Level (with x30 telescopic system), owned by John Barnard, for most surveys. It is used to determine summit and col positions and to measure drop. The closing error of our drop determinations (the difference between measuring drop from col to summit and from summit to col) averages 0.01m. Overall, the maximum expected error in a drop determination is ±0.028m which is lower than can be achieved by a survey-grade GNSS receiver for the same drop measurement. The estimated height uncertainty in locating the position of a col is frequently of the order of ±0.1m which dominates the overall measurement uncertainty for surveys of drop with level and staff. Individual legs in a line survey (the distance between level and staff) are typically 20m or less. The Leica NA730 is serviced by our local supply company, M&P Surveys, which ensures it meets its specification of 'an accuracy of 0.8mm over 30m'. The small closing errors and service support have always given us confidence in using the instrument for our surveys. We have therefore not sought to determine the measurement errors associated with use of the Leica NA730 hitherto, but these will be more significant when sighting over long distances.

Recently G&J Surveys carried out a survey of Currock Hill, a HuMP SW of Newcastle. LIDAR data published by the Environmental Agency shows two summits 530m apart differing in height by 0.044m. This difference is within the published error of the data. Part of the survey involved taking optical readings through the Leica NA730 between the two summits. Both forward (summit 1 to summit 2) and backward (summit 2 to summit 1) measurements were taken. The results confirmed the heights were very close. To establish the degree of confidence that the higher point was the true summit, we required an estimate of the measurement uncertainty. Any systematic error in the collimation of the level will cancel when forward and backward measurements are averaged, leaving a random error which is partially reduced by averaging. This prompted us to determine experimentally the random error associated with taking staff readings through the Leica NA730.

2) Experimental design

We planned to use a tape measure to mark two points 100m apart and set up an E staff supported by a tripod at each position. Survey bolts were placed in the ground at each of these points to prevent any vertical staff movement during the measurements. At a point mid-way between the two staffs, again determined by the tape measure, the level would be set up on a tripod and measurements taken of each staff. Then the level would be moved just half a metre or so at right angles to the line of the staffs and set up again and a new set of measurements taken. This procedure of moving and setting up the level around the central positon and taking new sets of readings would then be repeated ten times. The setup is shown in the diagram below.

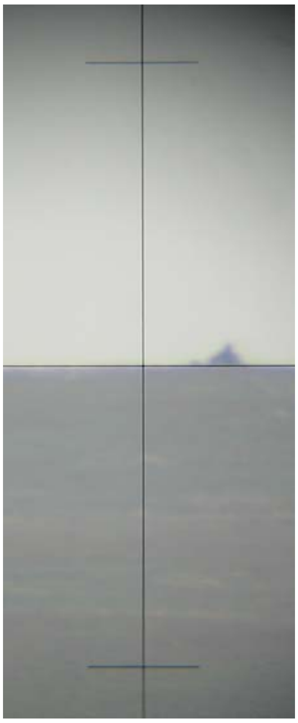


Finally, we planned to take staff readings with the level set up close to one staff and repeat with the level set up close to the second staff. These measurements would be used in conjunction with the dataset taken by the level at the mid-point to estimate of the systematic error in the level.

Conditions for the work, which took place between 14:00hr and 16:00hr GMT, were fair. The weather was mild, 10 degrees Celsius with negligible wind speed. Visibility was fair, although the weather was overcast.

3) Fieldwork

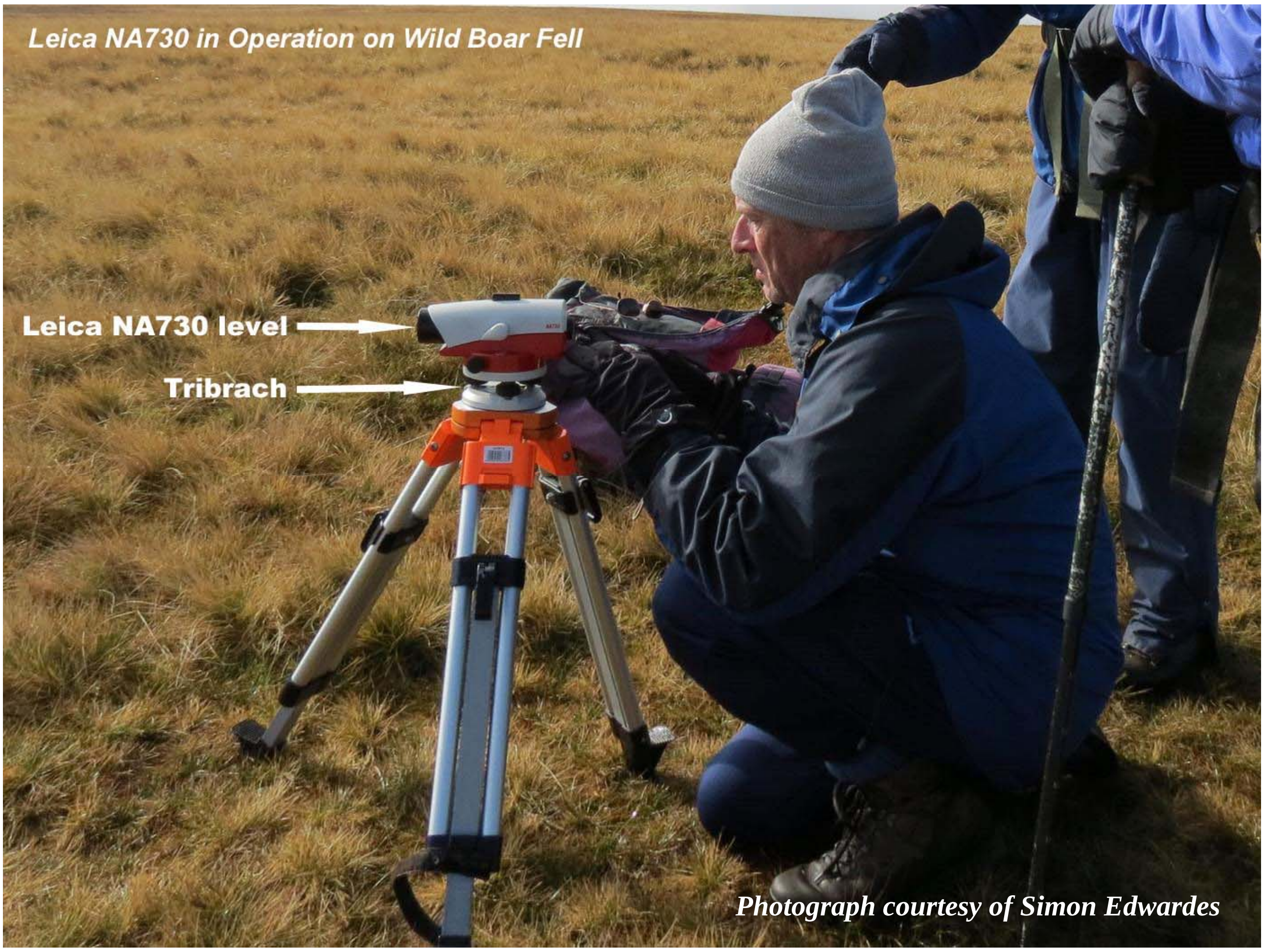
The site chosen for the work was a municipal playing field which gave an even and relatively flat surface upon which to work. The two staffs and the Leica NA730 level were set up as shown in the diagram above on the edge of the field. The orientation of the line between the two staffs was NE to SW. For data recording purposes (see worksheet in the Appendix) the SW staff was called the Backsight and the NE staff the Foresight. Three readings were taken from each staff, the central level line and the upper and lower stadia lines of the level. A typical view through the level showing these lines is given below (this photograph was taken on a different occasion).



The average of the three readings should be equal to the level line reading. If the difference is greater than 1mm, the three readings are repeated as it indicates that one of them is in error. This did not occur during the experiment. To check for operator bias, each of us recorded the first two sets of readings independently without knowledge of the other's readings. Since all six staff readings were identical, we concluded that the observer was not a significant source of error. Future sets of readings were taken alternately by each of us.

After each set of readings the level was moved a metre or so, as described above, and set up again. This entailed adjusting the legs of the tripod to obtain an approximately level position and then making fine adjustments of the tribrach until the bubble in the spirit level of the Leica NA730 was in its centre circle. By repeating this process, ten sets of readings were obtained. These are summarised in the Appendix.

Once completed, the next task was to set up the Leica NA730 level a few metres away from the NE staff and take readings of both the NE and SW staffs. Regrettably, the ground was not as level as anticipated and only centre line readings could be recorded, since either the lower or upper stadia line was below or above the staff. Finally, the Leica NA730 was moved to the SW staff and readings taken for it and the NE staff.



4) Results

The results of the fieldwork are tabulated in the Appendix. Columns 2 & 3 (Point Numbers 1 to 10) give the readings from the level central line for the backsight (SW) staff and foresight (NE) staff respectively (single wire levelling). The distance between level and each staff is 50m. Column 4 gives the height difference between them. This ranges from 0.812m to 0.816m with a standard deviation of 1.05mm.

Columns 11 & 12 give the mean of the horizontal, upper stadia and lower stadia readings, again for the backsight (SW) staff and foresight (NE) staff (triple wire levelling). This methodology is standard surveying practice. The height differences (column 13) range from 0.812m to 0.814m with a standard deviation of 0.76mm.

Columns 14 & 15 give the distance between the level and each of the staffs. This is calculated as 100 times the measured height difference between the stadia lines.

The bottom left corner of the table records the readings taken in the second phase when the level was set up near each staff in turn, so that the distance to the further staff was approximately 100m. The height differences range from 0.811 to 0.816m with a standard deviation of 2.2mm. Unlike the other measurements, this variation will include systematic error in the level and the effect of earth curvature on the foresight vs. backsight measurements.

5) Discussion

The standard deviation of 0.76mm is the estimate of the random error in a single level and staff measurement using triple wire levelling over 100m. For single wire levelling the corresponding statistic is 1.05mm. From the nature of the method, we expect the error to be proportional to distance.

The OS standard for the maximum expected error in a measurement is three times the standard deviation plus the systematic error. This is a "practically certain" interval giving over 99% confidence. The second phase of the experiment permits estimation of the systematic error. The effect of earth curvature is to decrease the height of the sighted object by 0.68mm over 100m, which is not negligible in the calculation (over 50m the correction is 0.17mm, and over the 20m leg of a typical line survey of drop it is only 0.03mm). The correction is applied by subtracting 0.68mm from the absolute height difference when sighting to the lower staff and adding 0.68mm when sighting to the higher staff. The systematic error calculated from the four corrected height differences is 1.1mm. A somewhat better estimate can be obtained by using the data from the first phase of the experiment to estimate the RMSE and the random component of the error. This gives a systematic error of 0.9mm. Note that the systematic error pertains to this particular level in its current state of calibration.

In practice, sightings are usually made in both directions and the two height differences are averaged. The systematic error and the correction for earth curvature both cancel, and the random error is reduced by a factor of $1/\sqrt{2}$. The standard error of the mean of the forward and backward determinations using triple wire levelling is therefore 0.53mm per 100m. Multiplying by 3 gives a maximum expected error of ±1.6mm per 100m.

On the Currock Hill survey that prompted the experiment, sightings between the two summits were taken over 500m in both directions using single wire levelling. The standard error of the mean is $5*1.05/\sqrt{2}$ = 3.7mm. The maximum error is obtained by multiplying this value by 3, giving ±0.011m. Note this is purely the measurement error associated with the level and staff measurements. To obtain the total error in the height difference between the two summits, a contribution should be added for the height uncertainty in locating the summit and col positions due to thickness of vegetation etc., and on this particular survey, an additional error to account for the difficulty in reading the staff accurately over such a long distance. The measured height difference was 0.05m, which is similar to the LIDAR figure but considerably more accurate.

6) Conclusions

The random error in a one-way measurement of height difference by an NA730 automatic level and staff using triple wire levelling is ±2.3mm per 100m of distance (three standard deviations). For the mean of forward and backward measurements the random error is ±1.6mm per 100m of distance. These values do not include errors associated with locating a summit or col position or placing the staff in unfavourable terrain.

The systematic error associated with the level over a distance of 100m was 1mm in this experiment. This error will disappear if an average of forward and backward measurements is taken.

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1 February 2017

Appendix

Title:- **Leica NA730 Evaluation - Chester Recreational Ground**

Instrument Leica NA730 **Date:-** 16-Jan-17

	Horizontal Line			Lower Stadia Line			Upper Stadia Line							
Point Number	**Backsight R**	**Foresight F**	**Height Diff**	**Backsight R**	**Foresight F**	**Height H**	**Backsight R**	**Foresight F**	**Height H**	**Mean BS**	**Mean FS**	**Height Difference**	**BS Distance**	**FS Distance**
	metres	**metres**	**metres**	**metres**	**metres**	**metres**	**metres**	**metres**	**metres**	**metres**	**metres**	**metres**	**metres**	**metres**
South to North														
1	1.351	0.538	0.813	1.100	0.288		1.600	0.788		**1.350**	**0.538**	0.812	50.000	50.000
2	1.465	0.651	0.814	1.215	0.400		1.715	0.901		**1.465**	**0.651**	0.814	50.000	50.100
3	1.339	0.523	0.816	1.085	0.270		1.590	0.778		**1.338**	**0.524**	0.814	50.500	50.800
4	1.376	0.562	0.814	1.125	0.310		1.628	0.815		**1.376**	**0.562**	0.814	50.300	50.500
5	1.289	0.474	0.815	1.037	0.221		1.538	0.727		**1.288**	**0.474**	0.814	50.100	50.600
6	1.413	0.599	0.814	1.161	0.349		1.662	0.851		**1.412**	**0.600**	0.812	50.100	50.200
7	1.342	0.530	0.812	1.090	0.276		1.595	0.781		**1.342**	**0.529**	0.813	50.500	50.500
8	1.369	0.555	0.814	1.116	0.301		1.620	0.808		**1.368**	**0.555**	0.814	50.400	50.700
9	1.172	0.358	0.814	0.918	0.109		1.425	0.609		**1.172**	**0.359**	0.813	50.700	50.000
10	1.420	0.606	0.814	1.170	0.355		1.670	0.857		**1.420**	**0.606**	0.814	50.000	50.200
N100m	1.675	0.859	0.816											
S100m	1.006	0.195	0.811											
S100m	1.031	0.219	0.812											
N100m	1.774	0.960	0.814											